# POLARISATION OF NUCLEONS DUE TO ANOMALOUS MAGNETIC MOMENT IN HIGHER BORN APPROXIMATION

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**ABSTRACT.** (1) We have extended Schwinger's calculation on polarisation of neutrons up to the second Born approximation taking non-relativistic form of the interaction of spin orbit coupling. (2) Considering the corresponding invariant interaction of the form $F_{\mu\nu}\,\gamma_\mu\,\gamma_\nu$ we have calculated the expression for the polarisation of the scattered particles in first order relativistic Born approximation. This result agrees with Wolfenstein's expression except for the characteristic Coulomb phase factor. (3) We have also made an attempt to calculate the corresponding second order relativistic Born approximation.

Mott (1929) has shown from Dirac's electron theory that electrons are polarised when they are scattered by a Coulomb field. A non-relativistic reduction of Dirac's theory shows that this polarisation is due to the spin orbit coupling in Coulomb field. Recently Schwinger (1948) has suggested a method for polarising neutrons by making use of the spin-orbit interaction of the magnetic moment of neutron with the Coulomb field of the scattering nucleus. The spin-orbit interaction of neutron moving in nuclear Coulomb field $V$ is given by the following contribution to neutron Hamiltonian

$$H' = \mu(e\hbar/2m^2c^2)(\sigma\,.\,c \times \vec{p})$$

where the electric field $\epsilon = -\Delta V$

and $\mu$ is the neutron magnetic moment in units of $(e\hbar/2mc)$, $\sigma$ is the Pauli spin vector, and $\vec{p}$ is the momentum of the neutron. Schwinger has made the non-relativistic calculations in the first Born approximation, we have here extended his result to second Born approximation.

Later Wolfenstein (1949) has investigated the polarisation of proton scattered by nuclear Coulomb field. He has introduced a term representing the interaction of the anomalous magnetic moment with the electromagnetic field of the invariant form proportional to $F_{\mu\nu}\,\gamma_\mu\,\gamma_\nu$ as first proposed by Pauli.

Wolfenstein has given an approximate expression for the scattered wave which is valid only when $v^2/c^2 << 1$ and $\alpha^2 = (Z/137)^2 << 1$ where $Z$ is the charge of target. He states that this expression can be obtained from Mott's expansion of the solution of the second order equation arising from the Dirac's linear equation modified by the addition of the Pauli term. Wolfenstein's formula is identical with that obtained in first Born's approximation taking the non-relativistic interaction term $\mu(e\hbar/2m^2c^2)\sigma.\epsilon\times\vec{p}$.

In this paper we propose to set up an integral equation corresponding to the modified Dirac's equation. Iterating the integral equation only once the scattered wave obtained by this method agrees with that obtained by Wolfenstein's method except for the characteristic Coulomb phase factor. Next we have made an attempt to extend the calculation by iterating the above mentioned integral equation twice which corresponds to second relativistic Born's approximation.

First we give the non-relativistic calculation of the scattered wave due to spin orbit interaction. We can write

$$\psi = \psi_0 + \psi_1 + \psi_2 + \cdots\cdots \qquad \ldots \quad (1)$$

$\psi_0$ is the incident wave $e^{ip_1 . r}\chi$ where $\chi$ is the spin wave function. $\psi_1$ and $\psi_2$ are respectively the first and second Born's approximations of the scattered wave. The calculation of $\psi_1$ is due to Schwinger and it is given by (taking $\hbar = 1$ and $c = 1$).

$$\psi_1 = \frac{1}{4\pi}\frac{e^{ipr}}{r}\left[\int e^{-ip_2 . r'}\, 2mH'(r')e^{ip_1 . r'}\, d^3r'\right]\chi(p_1)$$

$$= +\frac{e^{ipr}}{r} \cdot \frac{iZe^2\mu}{m} \cdot \frac{\sigma . p_2\times p_1}{|p_2-p_1|^2} \qquad \ldots \quad (2)$$

where $\vec{p}_1$ and $\vec{p}_2$ are respectively the momentum of initial and final states.* The expression for the second Born's approximation of the scattered wave is given by

$$\psi_2 = \frac{1}{4\pi}\frac{e^{ipr}}{r}\left[(2\pi)^{-3}\int\frac{d^3q}{q^2-(p^2+i\epsilon)}\int e^{ip_2 . r'}\; 2mH'(r')e^{iq . r'}\, d^3r'\right.$$

$$\left.\times\int e^{-iq . r''}\, 2mH'(r'')e^{ip_1 . r''}\, d^3r''\right]\chi(p_1)$$

$$= -\frac{e^{ipr}}{r}\frac{1}{2\pi^2}\left(\frac{Ze^2\mu}{m}\right)^2\int\frac{(\sigma . p_2\times q)(\sigma . q\times p_1)d^3q}{|p_2-q|^2|q-p_1|^2(q^2-(p^2+i\epsilon))}\,\chi(p_1)$$

$$= -\frac{e^{ipr}}{r}\frac{1}{2\pi^2}\left(\frac{Ze^2\mu}{m}\right)^2\left[i\int\frac{(\sigma . q)(q . p_1\times p_2)d^3q}{|p_2-q|^2|q-p_1|^2(q^2-(p^2+i\epsilon))}\right.$$

$$\left.+\int\frac{(q . p_2)(p_1 . q)-q^2(p_1 . p_2)}{|p_2-q|^2|q-p_1|^2(q^2-(p^2+i\epsilon))}\,\chi(p_1)\right.$$

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* The letters occuring in the scalar product, vector product and within modulus sign denote vector quantities. The rest are scalar quantities.

$$= \frac{e^{ipr}}{r} \frac{Z^2e^4\mu^2}{2m^2} p\, \sigma \,.\, \vec{n}\left(-\tan\theta/2\; ln \sin^2\theta/2 - i\pi \tan\theta/2 + i\pi \frac{\sin^2\theta/2}{\cos\theta/2}\right) \chi(p_1)$$

$$+ \frac{e^{ipr}}{r}\left(\frac{Ze^2\mu}{m}\right)^2 \left[p\frac{\sin^2\theta}{8}\left\{\frac{i}{\sin^2\theta/2\ \cos^2\theta/2}\right.\right.$$

$$\left.+\left(-\frac{1}{\cos^2\theta/2}+\frac{2}{\cos^4\theta/2}\right)\frac{1}{\sin\theta/2}\pi/2+\frac{2i}{\cos^4\theta/2}(ln \sin\theta/2 + i\pi/2)\right\}$$

$$\left. + \frac{p\cos\theta}{2}\left(\frac{2i}{\cos^2\theta/2} ln \sin\theta + \pi\frac{\sin\theta/2}{\cos^2\theta/2} - \frac{\pi}{\cos^2\theta/2}\right)\right]\chi(p_1) \qquad \ldots \quad (3)$$

where
$$p_2 \times p_1 = \vec{n}\, p^2 \sin\theta$$

For the case of electron the first Born's approximation does not give any polarisation effect since $\sigma$ dependent part is imaginary and $\sigma$ independent part is real Here polarisation is in second Born's approximation where part of the $\sigma$ dependent term is real. If we put $\mu = \frac{1}{2}$ real part of $\sigma$ dependent term of the above calculation agrees with that obtained by Mott

We now give the relativistic calculation for the Scattered wave due to the interaction of the anomalous magnetic moment of the scattered particle in the field of the target nucleus. The covariant form of modified Dirac's equation in the case where the vector potential is zero is given by

$$\left(\sum_{k=1}^{3}\gamma_k p_k - \gamma_4 E + m\right)\psi = -\gamma_4 e V\psi + (e/2m)(\mu/i)\gamma_4\gamma_k i F_{4k}\psi \qquad \ldots \quad (4)$$

Multiplying the above equation from the left by the operator $\gamma_4\,\alpha\,.\,p - \gamma_4 E - m$ we get

$$(\nabla^2 + p^2)\psi = (\gamma_4\,\alpha.\,p - \gamma_4 E - m)\left\{(-\gamma_4 e V + \frac{e}{2m}(\mu/i)\alpha.\epsilon)\psi\right\} \qquad \ldots \quad (5)$$

Knowing the Green's function of the above differential equation we obtain an integral equation the first approximation of which is the following :

$$\psi = e^{ip_1.r}u(p_1) + \frac{e^{ipr}}{4\pi r}\int e^{-ip_2.r'}(\gamma_4\alpha\, p - \gamma_4 E - m)\left\{\left(-\gamma_4 e V + \frac{e}{2m}(\mu/i)\alpha\epsilon\right)\right.$$

$$\left. e^{ip_1.r'}u(p_1)\right\} d^3r'$$

Now,

$$\iota \cdot p\left(\frac{\alpha \cdot r'}{r'^3} e^{ip_1 \cdot r'}\right) = -i\alpha \cdot \nabla\left(\frac{\alpha \cdot r'}{r'^3} e^{ip_1 \cdot r'}\right) = \frac{p_1 \cdot r' + i\sigma \cdot p_1 \times r'}{r'^3} e^{ip}$$

$$= \frac{\alpha \cdot r'\alpha \cdot p_1 + 2i\sigma \cdot p_1 \times r'}{r'^3} e^{ip_1 \cdot r'} \quad \ldots \quad (6)$$

$$(\gamma_4\alpha.p - \gamma_4 E - m)\left\{\frac{\alpha.r}{r^3} e^{ip_1.r} u(p_1)\right\} = \frac{\alpha.r}{r^3}(-\gamma_4\alpha.p_1 + \gamma_4 E - m)e^{ip_1.r}u(p_1)$$

$$+ \frac{2i\gamma_4\sigma.p_1 \times r}{r^3} e^{ip_1.r}u(p_1) = \frac{2i\gamma_4\sigma\, p_1 \times r}{r^3} e^{ip_1.r}u(p_1) \quad \ldots \quad (7)$$

also

$$-(\gamma_4\alpha.p - \gamma_4 E - m)\left\{\gamma_4 \frac{e}{r} e^{ip_1.r}u(p_1)\right\} = -\gamma_4 \frac{e}{r}(-\gamma_4\alpha.p_1 - \gamma_4 E \quad m)e^{ip_1.r}u(p_1)$$

$$-i\alpha.\left(\operatorname{grad}\frac{e}{r}\right) e^{ip_1.r}u(p_1) = 2E\frac{e}{r} e^{ip_1.r}u(p_1) - i\alpha.\left(\operatorname{grad}\frac{e}{r}\right) e^{ip_1.r}u(p_1) \quad \ldots \quad (8)$$

Remembering the above relations we have

$$\psi = e^{ip_1.r}u(p_1) + \frac{e^{ipr}Ze}{4\pi r}\left[\frac{e}{2m}(\mu/i)\frac{8\pi\gamma_4\sigma.p_1 \times p_2}{|p_2 - p_1|^2} + \frac{4\pi e}{|p_2 - p_1|^2}\alpha.(p_2 - p_1)\right.$$

$$\left. + \frac{2Ee.4\pi}{|p_2 - p_1|^2}\right] u(p_1) \quad \ldots \quad (9)$$

If the incident beam is in the $z$ direction we have

$$\alpha.(p_2 - p_1)u(p_1) = \alpha.(p_2 - p_1)\begin{pmatrix} A \\ B \\ -\frac{p_{1z}\, A}{E+m} \\ +\frac{p_{1z}\, B}{E+m}\end{pmatrix} = \begin{pmatrix} -\frac{\sigma.(p_2 - p_1)\sigma.p_1}{E+m}\chi \\ \\ \sigma.(p_2 - p_1)\chi \end{pmatrix} \quad \ldots \quad (10)$$

In determining polarisation we are interested in the large component of the spinor represented by $\chi$ and rewriting the expression for $\psi$ in terms of the large component of the spinor we have

$$\psi = e^{ip_1.r}\chi(p_1) + \frac{e^{ipr}}{r} Ze\left[-\frac{i\mu e}{2m}\sigma.n \cot\theta/2\right.$$

$$\left. - \frac{e}{E+m} \cdot \frac{1}{4p^2\sin^2\theta/2}\Big((p_2 - p_1).p_1 + i\sigma.(p_2 \times p_1)\Big) + \frac{Ee}{2p^2 \sin^2\theta/2}\right]\chi(p_1)$$

$$= e^{ip_1 \cdot r}\chi(p_1) + \frac{e^{ipr} Ze}{r}\left[-i\sigma \cdot n\left(\frac{\mu e}{2m} + \frac{1}{2}\,\frac{e}{E+m}\right)\cot\theta/2 + \frac{e}{2(E+m)} + \frac{Ee}{2p^2 \sin^2\theta/2}\right]\chi(p_1) \qquad \dots (11)$$

The spin dependent amplitude of the scattered wave agrees with that obtained by Wolfenstein if we put $E = m$ in our formula except for the characteristic Coulomb phase factor.

The contribution of anomalous magnetic moment to spin dependent part of the scattered wave is twice that due to the intrinsic spin of the nucleon.

For the case of electron the above expression for the scattered wave reduces to that obtained by Mott and is given by

$$\psi_{sc} = \frac{e^{ipr}}{r}\,\frac{Ze^2E}{2p^2}\left[\left(\operatorname{cosec}^2\theta/2 + \frac{m-E}{E}\right) + \frac{m-E}{E}\begin{pmatrix} 0 & -e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix}\right]\chi(p_1) \quad \dots (12)$$

where we have put

$$\sigma \cdot n = \frac{\sigma \cdot p_2 \times p_1}{p^2 \sin\theta} = i\begin{pmatrix} 0 & -e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix}, \ \varphi \text{ being the azimuthal angle.}$$

Iterating twice the integral equation corresponding to equation (5) the second Born approximation of the scattered wave is given by

$$\psi_2 = \frac{e^{ipr}}{4\pi r}\left(\frac{Ze^2\mu/i}{2m}\right)^2 \frac{1}{(2\pi)^3}\left[\int \frac{d^3q}{q^2-(p^2+i\epsilon)}\int e^{ip_2 \cdot r'}\left\{(-i\gamma.\Delta-\gamma_4E-m)\frac{\alpha \cdot r'}{r'^3}e^{iq \cdot r'}\right\}d^3r'\right.$$

$$\left.\times\int e^{-iq \cdot r''}\left\{(-i\gamma \cdot \Delta - \gamma_4 E - m)\frac{\alpha \cdot r''}{r''^3}e^{ip_1 \cdot r''}\right\}d^3r''\right]u(p_1) \quad \dots (13)$$

We insert the unit matrix $u(q)\bar{u}(q) = \sum_{r=1}^{r=4} u_r(q)\bar{u}_r(q) = I$ after $e^{iq \cdot r'}$ where spinor $u_r(q)$'s are the four solutions of the equation. $(\gamma \cdot q - \gamma_4 E(q) + m) = 0$. Hence

$$\psi_2 = \frac{e^{ipr}}{4\pi r}\left(\frac{Ze^2\mu/i}{2m}\right)^2 \frac{1}{(2\pi)^3}\left[\int \frac{d^3q}{q^2-(p^2+iE)}\cdot\frac{8\pi\gamma_4\sigma \cdot p_2 \times (p_2-q)}{|p_2-q|^2}\cdot\frac{8\pi\gamma_4\sigma \cdot q \times (q-p_1)}{|q-p_1|^2}u(p_1)\right]$$

$$+\int \frac{d^3q}{q^2-(p^2+i\epsilon)}\cdot\frac{\gamma_4E(q)-\gamma_4E}{1}\cdot\frac{-i4\pi\alpha.\,(p_2-q)}{|p_2-q|^2}\cdot\frac{8\pi\gamma_4\sigma \cdot p_1 \times (p_1-q)}{|q-p_1|^2}u(p_1) \quad \dots (14)$$

Here the first part of the wave $\psi_2$ called $\psi_2'$ agrees with $\psi_2$ of equation (3) calculated in non-relativistic Born approximation. In the second part of $\psi_2$ called $\psi_2''$ the integral involving $E(q)$ is zero since $E(q)$ will have both $+$ *ve* and $-$ *ve* signs. Then $\psi_2''$ is given by

$$\psi_2'' = +\frac{e^{ipr}}{4\pi r}\left(\frac{Ze^2\mu/i}{2m}\right)^2 \frac{1}{(2\pi)^3}\, 32\pi^2 iE \int \frac{d^3q}{q^2-(p^2+i\epsilon)}$$

$$\times \frac{q\cdot p_2\times p_1+i\sigma\cdot\{p_1(p_2\cdot q)-q(p_1\cdot p_2)\}-i\sigma\cdot\{p_1q^2-q(p_1\cdot q)\}}{|p_2-q|^2\;|q-p_1|^2}\,\rho_1 u(p_1) \qquad \text{... (15)}$$

Expressing again $\psi_2''$ in terms of the large component of spinor using the relation (10) we have

$$\psi_2'' = -\frac{e^{ipr}}{4\pi r}\left(\frac{Ze^2\mu/i}{2m}\right)^2 \frac{1}{(2\pi)^3}\frac{32\pi^2E}{E+m}\int \frac{d^3q}{(q^2-(p^2+i\epsilon))|p_2-q|^2|q-p_1|^2}$$

$$\times[p_1^2(p_2\cdot q)-(p_1\cdot q)(p_1\cdot p_2)-p_1^2q^2+(p_1\cdot q)(p_1\cdot q)-i\sigma\cdot(q\times p_1)(p_1\cdot p_2)$$

$$+i\sigma\cdot(q\times p_1)(p_1\cdot q)]\chi(p_1)$$

$$= -\frac{e^{ipr}}{4\pi r}\left(\frac{Ze^2\mu/i}{2m}\right)^2\frac{1}{(2\pi)^3}\cdot\frac{32\pi^2Ep}{E+m}\Big[\frac{\pi^2\sin^2\theta}{8}\Big\{4i\left(\frac{1}{\cos^2\theta/2}+\frac{1}{\cos^4\theta/2}\right)(ln\sin\theta/2+i\pi/2)$$

$$+\frac{2i}{\cos^2\theta/2}\left(2-\frac{1}{\sin^2\theta/2}\right)+\left(\frac{2}{\cos^4\theta/2}+\frac{1}{\cos^2\theta/2}\right)\frac{\pi}{\sin\theta/2}$$

$$-\frac{\pi}{\cos^2\theta/2}(\mathrm{cosec}\,\theta/2-1)-\frac{i}{\cos^2\theta/2}\,ln\sin^2\theta/2\Big\}-\pi^2\left(\frac{2i}{\cos^2\theta/2}\,ln\sin\theta/2\right.$$

$$\left.+\frac{\pi\sin\theta/2}{\cos^2\theta/2}-\frac{\pi}{\cos^2\theta/2}\right)\Big]\chi(p_1)$$

$$-\frac{e^{ipr}}{4\pi r}\left(\frac{Ze^2\mu/i}{2m}\right)^2\frac{1}{(2\pi)^3}\cdot\frac{32\pi^2Ep}{E+m}\,\sigma\cdot\vec{n}\Big[-\frac{i}{2}\sin\theta\cos\theta$$

$$\Big\{-\frac{\pi^3}{4\cos^2\theta/2}(\mathrm{cosec}\,\theta/2-1)$$

$$-\frac{\pi^2 i}{4\cos^2\theta/2}\,ln\sin^2\theta/2\Big\}-\frac{i\pi^2}{4}\cdot 2\sin\theta\cos\theta\Big\{i\left(\frac{2}{\cos^4\theta/2}+\frac{1}{\cos^2\theta/2}\right)(ln\sin\theta/2+i\pi/2)$$

$$+\frac{i}{\cos^2\theta/2}+\frac{\pi}{\cos^4\theta/2\,\sin\theta/2}\Big\}-\frac{i\pi^2}{4}\sin\theta(1-\cos\theta)\Big\{\frac{2i}{\cos^4\theta/2}$$

$$\left(ln\sin\theta/2+i\frac{\pi}{2}\right)$$

$$+\frac{i}{\cos^2\theta/2\sin^2\theta/2}+\left(-\frac{1}{\cos^2\theta/2}+\frac{2}{\cos^4\theta/2}\right)\frac{\pi/2}{\sin\theta/2}\Big\}\Big]\times\chi(p_1) \quad \ldots \quad (16)$$

The amplitude of the scattered wave is of the form

$$\psi_{sc}=\frac{e^{ipr}}{r}f(\theta)\chi(p_1) \quad \ldots \quad (17)$$

We split up $f(\theta)$ into $\sigma$-independent and $\sigma$ dependent components and write

$$f(\theta)=f_1(\theta)+\sigma\,.\,nf_2(\theta) \quad \ldots \quad (18)$$

$f_1(\theta)$ accounts for the amplitude of the wave scattered by nuclear forces and other types of forces independent of spin term.

The intensity of scattered wave is given by

$$r^2(\psi_{sc}\,,\psi_{sc})=|f_1(\theta)|^2+|f_2(\theta)|^2+2Re[f_1(\theta)f_2(\theta)]\,n\,.\,P_{inc} \quad \ldots \quad (19)$$

where $\qquad P_{inc}=(\chi\sigma\chi)$

Following the notation of Schwinger the vector $P$ represents the polarisation state of the beam. For an initially unpolarised beam for which $P_{inc}=(\chi\,\sigma\,\chi)=0$, polarisation state of scattered wave is given by

$$P_{sc}=\vec{n}\,\frac{2Re[f_1(\theta)f_2(\theta)]}{|f_1(\theta)|^2+|f_2(\theta)|^2}=n\,P(\theta) \quad \ldots \quad (20)$$

The polarisation of the scattered beam can be observed by subjecting once scattered polarised beam to a second scattering process. It has been shown that the left hand and right hand asymmetry of double scatterings is

$$R=\frac{1+P(\theta_1)+P(\theta_2)}{1-P(\theta_1)P(\theta_2)} \quad \ldots \quad (21$$

Of the integrals occurring in the paper, the first two have been evaluated by Dalitz and the third one is straight forward, though tedious. Their values are as follow :

$$\int\frac{d^3q}{|p_2-q|^2|q-p_1|^2(p^2-q^2+i\epsilon)}=\int\int_{-1}^{1}\frac{d^3q\quad dZ}{[|q-P|^2+\Lambda^2]^2(p^2-q^2+i\epsilon)}$$

$$\lambda\to o$$

$$=I$$

$$\int\frac{q_r d^3q}{|p_2-q|^2|q-p_1|^2(p^2-q^2+i\epsilon)}=\int\int_{-1}^{1}\frac{q_r\quad d^3q\quad dZ}{[|q-P|^2+\Lambda^2]^2(p^2-q^2+i\epsilon)}$$

$$\lambda\to o$$

$$=\frac{(p_1+p_2)_r}{2}\left\{-\frac{\pi^3}{4p^3\cos^2\theta/2}(\operatorname{cosec}\theta/2-1)-\frac{\pi^2 i}{4p^3\cos^2\theta/2}\ln\sin^2\theta/2+I\right\}$$

$$\int\frac{q_r\,q_s\,d^3q}{|p_2-q|^2\,|q-p_1|^2\,(p^2-q^2+i\epsilon)}=\int\int_{-1}^{1}\frac{q_rq_s\,d^3q\,dZ}{[\,|q-P|^2+\Lambda^2]^2(p^2-q^2+i\epsilon)}$$

$$\lambda\to 0$$

$$=\int_{-1}^{1}\Bigg[\frac{\pi^2\delta_{rs}}{2P}\left(\frac{\Lambda+ip}{P}+i\frac{p^2+P^2+\Lambda^2}{2P^2}\ln\frac{p-P+i\Lambda}{p+P+i\Lambda}\right).$$

$$\lambda\to 0$$

$$-\frac{\pi^2p^2}{8}\left\{(p_1+p_2)_r(p_1+p_2)_s+(p_1-p_2)_r(p_1-p_2)_s+(Z^2-1)(p_1-p_2)_r(p_1-p_2)_s\right\}$$

$$\times\left\{\frac{3i}{2}\left(\frac{P^2+p^2+\Lambda^2}{P^5}\right)\ln\frac{p-P+i\Lambda}{p+P+i\Lambda}+\frac{2(\Lambda+ip)}{P^4}-\frac{2}{\Lambda(p+P+i\Lambda)(p-P+i\Lambda)}\right.$$

$$\left.+\frac{i(p^2+3P^2+\Lambda^2)}{2P^4}\left(\frac{1}{p-P+iV}+\frac{1}{p+P+i\Lambda}\right)\right\}\Bigg]dZ$$

$$=\frac{\pi^2\delta_{rs}}{2}\left(\frac{2i}{p\cos^2\theta/2}\ln\sin\theta/2+\frac{\pi}{p}\frac{\sin\theta/2}{\cos^2\theta/2}-\frac{\pi}{p\cos^2\theta/2}\right)$$

$$-\frac{\pi^2}{8}\left\{p_1+p_2)_r(p_1+p_2)_s+(p_1-p_2)_r(p_1-p_2)_s\right\}\frac{2}{p^3}\left[i\left(\frac{2}{\cos^4\theta/2}+\frac{1}{\cos^2\theta/2}\right)\right.$$

$$\left.(\ln\sin\theta/2+i\pi/2)+\frac{i}{\cos^2\theta/2}+\frac{2}{\cos^4\theta/2\sin\theta/2}\cdot\frac{\pi}{2}-\frac{I'}{\pi^2}\right]$$

$$+\frac{\pi^2}{8p^3}(p_1-p_2)_r(p_1-p_2)_s2\left[\frac{2i}{\cos^4\theta/2}\left(\ln\sin\theta/2+\frac{i\pi}{2}\right)+\frac{i}{\sin^2\theta/2\cos^2\theta/2}\right.$$

$$\left.+\left(-\frac{1}{\cos^2\theta/2}+\frac{2}{\cos^4\theta/2}\right)\frac{\pi/2}{\sin\theta/2}\right]$$

where $\vec{P}=\frac{1}{2}[(1+Z)\vec{p}_1+(1-Z)\vec{p}_2]$, so that $P^2=p^2(\cos^2\theta/2+Z^2\sin^2\theta/2)$ and $\Lambda^2=\lambda^2+p^2\sin^2\theta/2\ \ (1-Z^2)$

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